

Equations - Mechanics

Kinematics

Linear

$$s = s_0 + v_0t + \frac{1}{2}at^2$$

$$v_f = v_0 + at$$

$$v_f^2 = v_0^2 + 2ad$$

$$d \text{ (range)} = v_0^2 \sin(2\theta)/g$$

$$h_{\max} = \sin^2\theta v_0^2 / 2g$$

Tang velocity

Centripetal accel

Tang accel

Rotational

$$\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$$

$$\omega_f = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \Delta\omega/t$$

$$v_t = \omega r$$

$$a_c = v_t^2/r = \omega^2 r$$

$$a_t = \alpha r$$

Force/work

$$F = ma = Gm_1m_2/r^2$$

$$p = mv = Ft$$

$$J = Ft = \Delta p$$

$$W = Fd = \Delta E$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 = mgh = Fd = \frac{1}{2}p^2/m$$

$$P = E/t = Fv$$

$$F_g = Gm_1m_2/r^2$$

$$[v_{f1} = v_{i1}(m_1 - m_2)/(m_1 + m_2)]$$

$$v_{\text{term}} = \sqrt{2mg/C\rho A}$$

$$\tau = rF\sin\theta = I\alpha$$

$$L = I\omega = rp \quad \text{[only around fixed axis or axis of symmetry]}$$

$$\Delta L = r\Delta p = \tau t = Fsr$$

$$= \tau\theta$$

$$= \tau\theta, \frac{1}{2}I\omega^2$$

$$= \tau\theta t = \tau\omega$$

$$F_{\text{drag}} = \frac{1}{2}C\rho Av^2$$

$$v_{f2} = v_{i1}(2m_1)/(m_1 + m_2)$$

$$T = 2\pi\sqrt{r/g} \text{ (orbital period)}$$

2 obj collision, w obj 2 at rest]

Moment of inertia of various objects

Object	Center	Edge/End
Thin Rod	$1/12 ML^2$	$1/3 ML^2$
Plane/slab	$1/12 Ma^2$	$1/3 Ma^2$
Cylinder/solid	$\frac{1}{2} MR^2$	(hoop) MR^2
Sphere/solid	$2/5 MR^2$	(hollow) $2/3 MR^2$
General	cMR^2	
Rolling Downhill	$v_{\text{cm}} = \sqrt{2gh / (1+c)}$	$a_{\text{cm}} = g \sin\theta / (1+c)$

Parallel Axis Theorem

$$I = I_{\text{cm}} + Md^2$$

M = Mass

R = Radius

L = Length

a = area

d = distance

I = moment of inertia

cm – center mass

SHM

$$f = 1/T = (1/2\pi) \sqrt{k/m}$$

$$T = 2\pi\sqrt{m/k}$$

$$\omega = d\phi/dt = 2\pi f = 2\pi/T$$

$$\phi = \omega t + \phi_0$$

$$x(t) = A \cos(2\pi t/T) = A \cos(\omega t + \phi_0) = A \cos(2\pi ft)$$

$$\Delta\phi = \omega t$$

$$\mu = \frac{T}{v^2}$$

$$v(t) = dx/dt = -\omega A \sin(\omega t + \phi_0) = -v_{\max} \sin(2\pi t/T) = -v_{\max} \sin(2\pi ft)$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$a(t) = d^2x/dt^2 = -\omega^2 A \cos(\omega t + \phi_0)$$

$$a_x = -\omega^2 x = -k/m x$$

$$u = Ae^{-v/v_0}$$

Legend

s, r, d or h = distance or radius or height

v = velocity

a = acceleration

a_t = tangential acceleration

F = force

Q, q = charge (C)

E = Energy; electric field (J; N/C or V/m)

U = potential energy

J = Joules (Nm or kg m²/s²)

μ = linear density (kg/m); permeability Const ($4\pi \times 10^{-7} \text{Tm/A}$)

ϵ = permittivity const ($8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$)

C = capacitance (F)/Coulomb (C)

θ, Φ = angle

ω = angular velocity

α = angular acceleration

a_c = centripetal acceleration

τ = torque

λ = charge/length

W = work (J)

K = Kinetic Energy (J)

u_E = elec energy density (J/m³)

$K = 1/2\pi\epsilon_0 (8.99 \times 10^9 \text{Nm}^2/\text{C}^2)$

V = voltage or electrical pot. (V)

A = area

f = frequency

g = acceleration of gravity

G = gravitational constant

T = period; tension (N)

η = charge/area

Φ = electric flux (Vm or Nm²/C)

P = power (W; J/s)

ρ = charge density; (Q/m³)

κ = dielectric constant

p = dipole moment (Cm or D)

Identities

$x = \cos\theta$ $y = \sin\theta$ $\theta = \tan^{-1} y/x$

$\tan\theta = \sin\theta / \cos\theta$

$\sin 2\theta = 2\sin\theta \cos\theta$

$p = sq$

Other

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Constants

$$\epsilon = 8.55 \times 10^{-12} \text{C}^2/\text{Nm}^2 \quad K = 1/2\pi\epsilon = 8.99 \times 10^9 \text{Nm}^2/\text{C}^2$$

$$G = 6.67 \times 10^{-11} \text{N} \quad (q)e^- = 1.6 \times 10^{-19} \text{C} \quad (m)e^- = 9.11 \times 10^{-31} \text{kg} \quad (m)p^+ = 1.67 \times 10^{-27} \text{kg}$$