## Equations of Electricity and Magnetism

( $R$ is radius, $r$ is distance)
Point/sphere(ext)
Linear
Ring
Dipole
Disk
Planes, 2, between
$\mathrm{E}=\frac{\mathrm{V}}{\mathrm{s}}$
$\mathrm{E}_{\text {solenoid_inside }}=\mathrm{r} / 2\left|\frac{\delta B}{\delta t}\right|$

Finite
$E=K q / r^{2}$
$E=K O / r V\left(r^{2}+L^{2} / 2\right)$
$E=K r Q /\left(r^{2}+R^{2}\right)^{3 / 2}$
$E=F / q=K 2 p / r^{3}=K s q / r^{3}$
$E=\left(\eta / 2 \varepsilon_{0}\right)\left(1-\left(r / V\left(r^{2}+R^{2}\right)\right.\right.$
$\mathrm{E}=\mathrm{Q} / \mathrm{A} \varepsilon_{0}=\eta / \varepsilon_{0}$
$\mathrm{E}_{\mathrm{c}}=\mathrm{Q}_{\mathrm{c}} /\left(\mathrm{A}_{\mathrm{c}} \varepsilon_{0}\right)$
$\mathrm{E}_{\text {solenoid_outside }}=\mathrm{R}^{2} / 2 \mathrm{r}\left|\frac{\delta B}{\delta t}\right|$

Infinite
$2 K \lambda / r$
-
$\tau=\sin \theta s q E=\sin \theta p E$
$\eta / 2 \varepsilon_{0}$

Electrical Potential
$\mathrm{V} \equiv \mathrm{Es}=\mathrm{U}_{\mathrm{q}} / \mathrm{q}^{\prime}=\mathrm{KQ} / \mathrm{r}=-\mathrm{L} \mathrm{dI} / \mathrm{dt} \quad \mathrm{V}_{\mathrm{c}}=\mathrm{Q} \Delta \mathrm{x} /\left(\varepsilon_{0} \mathrm{~A}\right)$ Units: $(1$ volt $=1 V \equiv 1 \mathrm{~J} / \mathrm{C})(1 \mathrm{~N} / C=1 \mathrm{~V} / \mathrm{m})(F=C / V)$
$\varepsilon=\Delta \Phi_{\mathrm{m}} / \Delta \mathrm{t}=\oint E d s=-\frac{\delta}{\delta t} \int_{A} \vec{B} d \vec{A}=-\mathrm{L} \mathrm{dl} / \mathrm{dt}$
Force
$\vec{F}=\mathrm{q} \vec{v} \times \vec{B}=\mathrm{q} v \mathrm{~B} \sin \theta=\mid \vec{l} \overrightarrow{w i r e} \mathrm{X} \vec{B}=\mathrm{IlB}=\mathrm{v}^{2} \mathrm{~B}^{2} / \mathrm{R} \quad \quad \vec{F}_{\text {parallelWires }}=\mathrm{ll}_{1} \mathrm{~B}_{2}=\mu_{0} \mathrm{ll}_{1} \mathrm{l}_{2} / 2 \pi \mathrm{~d}$

## Energy

Energy $=1 / 2 \mathrm{mv}^{2}=1 / 2 k x^{2}=1 / 2 \mathrm{CV}_{\mathrm{c}}{ }^{2}=1 / 2 \mathrm{LI}^{2}=\mathrm{q} \Delta \mathrm{V}(\mathrm{J}) \quad$ Energy density $u=\mathrm{U}_{\mathrm{c}} / \mathrm{Ad}=1 / 2 \kappa \varepsilon_{0} \mathrm{E}^{2}=\mathrm{B} / 2 \mu_{0}\left(\mathrm{~J} / \mathrm{m}^{3}\right)$
$\Delta \mathrm{U}_{\mathrm{E}}=\mathrm{qEs} \mathrm{s}_{\mathrm{f}}-\mathrm{qE} s_{\mathrm{i}}=-\mathrm{W}_{\text {elec }}=\mathrm{U}_{0}+\mathrm{qEs} \quad \mathrm{U}_{\mathrm{E}}=\mathrm{q} \Delta \mathrm{V}=-\int \mathrm{F}_{\text {1on2 }} \mathrm{dx}$
$\mathrm{U}=-\mathrm{Kq}_{1} \mathrm{q}_{2} / \mathrm{r}=-\int K q_{1} q_{2} / x^{2} \mathrm{dx}=K q_{1} q_{2} / \mathrm{x}_{\mathrm{f}}-K q_{1} q_{2} / \mathrm{x}_{\mathrm{i}}$
$\mathrm{U}_{\text {cap }}=\mathrm{Q}^{2} / 2 \mathrm{C}=1 / 2 \mathrm{CV}_{\mathrm{c}}{ }^{2}=\kappa \varepsilon 0 \frac{A}{2} \bullet \mathrm{E}^{2} \mathrm{~d} \quad \mathrm{P}=\mathrm{U} / \mathrm{t}=\mathrm{I}^{2} \mathrm{R}=\mathrm{V}^{2} / \mathrm{R}=\mathrm{I} \Delta \mathrm{V}=1 / 2 \mathrm{LI}^{2} / \mathrm{t}=\mathrm{v}^{2} \mathrm{l}^{2} \mathrm{~B}^{2} / \mathrm{R}$
$\mathrm{U}_{\text {dipole }}=-\mathrm{pE} \cos \phi=-\vec{p} \cdot \vec{E}$

$$
\Delta U_{\text {elec }}=-W_{\text {elec }} \text { if } U_{\text {elec }}=K q_{1} q_{2} / r
$$

Electrical
$\mathrm{C}=\kappa \varepsilon_{0} \frac{A}{d}=\frac{Q}{V}$

$$
\begin{array}{ll}
\mathrm{V}=\frac{Q}{C}=\mathrm{IR}=\mathrm{v} \mathrm{LB}=\mathrm{L} \frac{\delta I}{\delta t} & \mathrm{~L}_{\text {solonoid }}=\mu_{\mathrm{o}} \mathrm{~N}^{2} \mathrm{~A} / \mathrm{I}=\Phi_{\mathrm{m}} / \mathrm{l} \\
\mathrm{I}=\mathrm{Q} / \mathrm{t}=\mathrm{v} \mathrm{LB} / \mathrm{R} & \mathrm{E}=\mathrm{V} / \mathrm{L}_{\text {wire }} \\
\mathrm{V}_{\mathrm{d}}=\mathrm{eE} \tau / \mathrm{m} & \tau=\mathrm{V}_{\mathrm{d}} \mathrm{~m} / \mathrm{eE}=\mathrm{m} / \mathrm{n}_{\mathrm{e}} \mathrm{e}^{2} \rho \\
\rho=\mathrm{RA} / \mathrm{L}=\sigma^{-1}=\mathrm{m} / \mathrm{n}_{\mathrm{e}} \mathrm{e}^{2} \tau & \mathrm{~J}=\mathrm{I} / \mathrm{A}=\mathrm{n}_{\mathrm{e}} \mathrm{ev}_{\mathrm{d}}=\sigma \mathrm{E} \\
\varepsilon=\Delta \Phi_{\mathrm{m}} / \Delta \mathrm{t}=\oint E d s=-\frac{\delta}{\delta t} \int_{A} \vec{B} d \vec{A}=-\mathrm{Ld} \mathrm{dl} / \mathrm{dt}
\end{array}
$$

$\mathrm{Q}=\mathrm{C} \Delta \mathrm{V}_{\mathrm{c}}=\kappa \varepsilon_{0} \frac{A}{d} \cdot \Delta \mathrm{~V}_{\mathrm{c}}=\mathrm{eN} \mathrm{N}_{\mathrm{e}}$
$N_{e}=i_{e} t=n_{e} A x=n_{e} A v_{d} t$
$\mathrm{R}=\rho \mathrm{L} / \mathrm{A}=\mathrm{V} / \mathrm{I}$
$\mathrm{P}=\mathrm{U} / \mathrm{t}=\mathrm{I}^{2} \mathrm{R}=\mathrm{V}^{2} / \mathrm{R}=\mathrm{I} \Delta \mathrm{V}=1 / 2 \mathrm{LI}^{2} / \mathrm{t}$
RC Cap discharge $\quad I=I_{0} e^{-\Delta t / R C}$
$\Delta V_{C}=\left(\Delta V_{C}\right)_{o} \mathrm{e}^{-\Delta t / R C}$
$\mathrm{Q}=\mathrm{Q}_{0}\left(\mathrm{e}^{-\Delta \mathrm{t} / \mathrm{RC}}\right) \quad \tau=\mathrm{RC}=\mathrm{L} / \mathrm{R}$
RC Cap charge
$\mathrm{I}=\mathrm{I}_{0} \mathrm{e}^{-\Delta \mathrm{t} / \mathrm{RC}}$
$\Delta V_{C}=V_{\text {battery }}\left(1-\mathrm{e}^{-\Delta t / R C}\right)$
$Q=Q_{0}\left(1-e^{-\Delta t / R C}\right)$

## Magnetic

$B_{\text {point }}=\mu_{0} q v \sin \theta / 4 \pi r^{2}$ (dir RHrule) $\quad B_{\text {wire }}=\mu_{0} I / 2 \pi r$ (dir RHrule) $\quad B_{\text {loop }}=\mu_{0} N I / 2 R$ (N-Num loops,R-radius)
$\vec{B}_{\text {dipole }}=2 \mu_{0} \vec{\mu} / 4 \pi z^{3}$ (on axis of mag moment) $\quad B_{\text {solonoid }}=\mu_{0} \mathrm{NI} / \mathrm{l}=\mu_{0} \mathrm{nl}(\mathrm{n}=\mathrm{N} / \mathrm{l}) \quad \oint B d s=\mu_{0} \mathrm{I}$
$\tau_{\text {loop }}=$ NIAB $\sin \theta\left(\begin{array}{c}\text { a area, } \theta \text { angle to loop plane }\end{array}\right)$
$\tau=\vec{\mu} \times \vec{B}=\mu B \sin \theta$
$\vec{\mu}=\mathrm{IA}=4 \pi \mathrm{z}^{3} \mathrm{~B} / 2 \mu_{0}$
$P=v^{2} l^{2} B^{2} / R=V^{2} / R=I^{2} R$
$\Phi_{\mathrm{m}}=\vec{A} \times \vec{B}=\mathrm{AB} \cos \theta=\mu_{0} \mathrm{I} \quad \Phi_{\mathrm{E}}=\frac{Q}{\epsilon_{0}}$
$\omega=\sqrt{1 / L C}$

Maxwell's Equations
$\begin{array}{lll}\text { Gauss's Law } & \vec{\nabla} \cdot \vec{E}=\frac{\rho}{\epsilon_{0}} \quad \vec{\nabla} \cdot \vec{G}=\hat{l} \frac{\delta G}{\delta x}+\hat{\jmath} \frac{\delta G}{\delta y}+\hat{k} \frac{\delta G}{\delta z} & \oint_{A} \vec{E} d \vec{A}=\frac{Q}{\epsilon_{0}} \\ \vec{\nabla} \cdot \vec{B}=0 & \oint_{A} \vec{B} d \vec{A}=0 \\ \text { Faraday's Law } & \vec{\nabla} x \vec{E}=-\frac{\delta \vec{B}}{\delta t} & \oint_{L} \vec{E} d \vec{L}=-\frac{\delta}{\delta t} \int_{A} \vec{B} d \vec{A} \\ \text { Ampere's Law } & \vec{\nabla} \times \vec{B}=\mu_{0} \vec{J}+\frac{1}{C^{2}} \frac{\delta \vec{E}}{\delta t}=\mu_{0}\left(\vec{J}+\epsilon_{0} \frac{\delta \vec{E}}{\delta t}\right) & \oint_{L} \vec{B} d \vec{L}=\mu_{0}\left(I+\epsilon_{0} \frac{\delta}{\delta t} \int_{A} \vec{E} d \vec{A}\right)\end{array}$

## Legend

| $\mathrm{s}, \mathrm{r}, \mathrm{d} \mathrm{L}$ or $\mathrm{h}=$ distance, radius or height(m) | $\theta, \Phi$ = angle | $\mathrm{A}=$ area; $\mathrm{ampere}^{\text {( }} \mathrm{C} / \mathrm{s}$ ) |
| :---: | :---: | :---: |
| $\mathrm{v}=$ velocity | $\omega=$ angular velocity (Rad/s) |  |
| $\mathrm{a}=$ acceleration | $\alpha=$ angular acceleration | $\mathrm{g}=$ acceleration of gravity |
| $\mathrm{at}_{\text {t }}=$ tangential acceleration | $\mathrm{ac}_{\mathrm{c}}=$ centripetal acceleration | $\mathrm{G}=$ gravitational constant |
| $\mathrm{F}=$ force; Farad $=\mathrm{C}^{2} \mathrm{~s}^{2} /\left(\mathrm{kg} \mathrm{m}^{2}\right)$ | $\mathrm{p}=$ dipole moment ( Cm or D ) | $\tau=$ torque( Nm ) |
| $\mathrm{Q}, \mathrm{q}=$ charge (C) | $\lambda$ = charge/length | $\eta$ = charge/area |
| $\mathrm{E}=$ electric field ( $\mathrm{N} / \mathrm{C}$ or $\mathrm{V} / \mathrm{m}$ ) | $\mathrm{W}=$ work ( J$)$; watts ( $\mathrm{J} / \mathrm{s}$ ) |  |
| $\mathrm{U}=$ potential energy ( J$)$ | $\mathrm{K}=$ Kinetic Energy ( J ) | $\mathrm{P}=$ power ( W ; $\mathrm{J} / \mathrm{s}$ ) |
| $\rho=$ charge density ( $\mathrm{C} / \mathrm{m}^{3}$ ); resistivity ( $\Omega \mathrm{m}$ ) | $\mathrm{u}_{\mathrm{E}}=$ elec energy density ( $\mathrm{J} / \mathrm{m}^{3}$ ) | $\mathrm{R}=$ resistance ( $\Omega$ ) |
| $\mathrm{n}_{\mathrm{e}}=$ conduction $\mathrm{e}^{-}$density | $\mathrm{N}_{\mathrm{e}}=\#$ e pass cross-section/s | $\mathrm{i}_{\mathrm{e}}=$ electron current (\# $\mathrm{e}^{-/ \mathrm{s}}$ ) |
| $\mathrm{v}_{\mathrm{d}}=$ drift velocity | $\sigma=$ conductivity | $B=$ magnetic field strength ( $\mathrm{T}, \mathrm{kg} / \mathrm{As}^{2}$ ) |
| $\mathrm{J}=$ current dens( $\mathrm{A} / \mathrm{m}^{2}$ ); Joules ( Nm or $\left.\mathrm{kg} \mathrm{m}^{2} / \mathrm{s}^{2}\right)$ | $\mathrm{e}=$ charge of electron | $\vec{\mu}=$ mag dipole moment vector ( $\mathrm{Am}^{2}$ ) |
| $\mathrm{V}=$ voltage or elect pot. ( V ); volume $\left(\mathrm{m}^{3}\right)$ | C = capacitance (F); Coulomb (C) | L = Inductance ( $\mathrm{H}, \mathrm{Wb} / \mathrm{A}, \mathrm{Tm}^{2} / \mathrm{A}$ ) |
| $\tau=$ avg t bet collisions(s); time const | $\Phi_{m}=$ Magnetic Flux ( Wb or $\mathrm{Tm}^{2}$ ) | $\Phi_{\mathrm{m}}=$ Electric flux ( $\mathrm{Vm} ; \mathrm{Nm}^{2} / \mathrm{C}$ ) |
| $\mathrm{T}=$ tesla (kg/As $\left.{ }^{2}, \mathrm{~N} / \mathrm{Am}\right)$ | $\underline{\mathrm{Wb}}=$ Weber $\left(\mathrm{kgm}^{2} / \mathrm{As}^{2}, \mathrm{Nm} / \mathrm{A}, \mathrm{Tm}^{2}\right)$ |  |


| $\epsilon=$ permittivityConst $\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right) ; e m f$ | $K=1 / 2 \pi \varepsilon_{0}\left(8.99 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}\right) \quad \kappa=$ dielectric constant |
| :--- | :--- |
| $\mu_{0}=$ permeability Const $\left(4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}\right)$ | $G=6.67 \times 10^{-11} \mathrm{~N}$ |
| (q) $e^{-}=1.6 \times 10^{-19} \mathrm{C}(\mathrm{m})=9.11 \times 10^{-31} \mathrm{~kg}$ | $(\mathrm{~m}) p^{+}=1.67 \times 10^{-27} \mathrm{~kg}$ |

## Other

$\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

