

Equations – P-Chem

GASES

Gas laws

$$\text{atm} = 1.01325 \text{ bar} = 101325 \text{ Pa}$$

$$\text{atm} = 760 \text{ torr}$$

$$\text{bar} = 1.0 \times 10^5 \text{ Pa}$$

$$\text{torr} = \text{mmHg} = 133.322 \text{ Pa}$$

Ideal gas law

$$pV = nRT = \frac{m}{M}RT \quad (p \text{ Pa, } V \text{ m}^3, T \text{ K})$$

$$pM = \rho RT \quad (M \text{ mole wt, } \rho \text{ g/dm}^3)$$

van der Waals law

$$p = \frac{nRT}{V-nb} - \frac{an^2}{V^2} \quad p_r = \frac{8T_r}{3V_r-1} - \frac{3}{V_r^2}$$

$$p = \frac{RT}{V_m-b} - \frac{a}{V_m^2} \quad [V_m \text{ molar vol} = \frac{V}{n}]$$

Berthelot

$$p = \frac{nRT}{V-nb} - \frac{an^2}{TV^2} \quad p_r = \frac{8T_r}{3V_r-1} - \frac{3}{T_r V_r^2}$$

Dieterici

$$p = \frac{RT e^{-\frac{na}{RTV}}}{V-nb} \quad p_r = \frac{T_r e^{2(1-\frac{1}{V_r T_r})}}{2V_r-1}$$

Virial

$$p = \frac{nRT}{V} \left\{ 1 + \frac{nB(T)}{V} + \frac{n^2 C(T)}{V^2} \right\}$$

quant reduced form $X_r = X/X_c$

molar fraction

$$x_j = \frac{n_j}{n}; n_j = nx_j$$

Kinetic model

Collision cross sectional area

$$\sigma = \pi d^2$$

$$\langle V_x^2 \rangle = \langle V^2 \rangle / 3$$

Mean square speed

$$\langle v^2 \rangle = \frac{3RT}{M} = \frac{3kT}{m}$$

Root mean square

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

Mean speed

$$\langle v \rangle = v_{\text{rms}} (8/3\pi)^{1/2} = (8RT/M\pi)^{1/2}$$

Most probable speed

$$v_{\text{mp}} = v_{\text{rms}} (2/3)^{1/2} = \sqrt{\frac{2RT}{M}}$$

Relative mean speed

$$v_{\text{rel}} = \langle v \rangle \sqrt{2} = v_{\text{rms}} \sqrt{\frac{16RT}{3\pi}} = \sqrt{\frac{16RT}{\pi M}} \quad (\text{identical molecules})$$

$$\mu = \frac{M_A M_B}{M_A + M_B}$$

$$v_{\text{rel}} = \sqrt{\frac{8RT}{\pi \mu}} \quad (\text{non-identical molecules})$$

Mean translational energy

$$\langle E_{\text{trans}} \rangle = \frac{mv^2}{2} = \frac{3}{2} pV = \frac{3}{2} RT$$

Collision Frequency

$$z = \sigma v_{\text{rel}} N = \frac{\sigma v_{\text{rel}} p}{kT} \quad (\text{particles}) = \frac{p N_A}{2\pi MRT} \quad (\text{walls/container})$$

Mean free path

$$\lambda = v_{\text{rel}} / z = \frac{kT}{p\sigma} \quad (\text{other atoms static}) = \sqrt{2} \frac{kT}{p\sigma} \quad (\text{all atoms dynamic})$$

Compression factor

$$Z = \frac{V_m}{V_m^0} = \frac{pV_m}{RT} = \frac{pV}{nRT}$$

Boyle Temperature

$$T_B = \frac{a}{bR} = \frac{27}{8} T_C$$

Effusion rate

$$\frac{\text{rate}_a}{\text{rate}_b} = \sqrt{\frac{M_b}{M_a}}$$

Constants

R = Gas constant = $N_A k$ (8.31446 J/Kmol or Pa m³/K mol) = 0.0831445 (dm³ bar /K mol)
 = (0.0820574 dm³ atm /K mol) = (62.364 dm³ Torr/K mol) = (1.98721 cal/K mol)
 k = Boltzmann's constant (1.38065x10⁻²³ J/K)

Legend

d = distance, Collision diam	A = area	σ = collision cross-sectional area
V = volume	V_m = molar volume (V/n)	V_m^0 = molar vol of ideal gas
v = velocity/speed	v_{rms} = root mean square velocity	$\langle v \rangle = v_{mean}$ = mean speed
v_{mp} = most probable speed	v_{rel} = relative mean speed	$\langle E_{trans} \rangle$ = mean translational energy
z = collision frequency	λ = mean free path	
E = Energy; electric field (J; N/C or V/m)	J = Joules (Nm or kg m ² /s ²)	
M = molar mass (Kg/mol)	F = force (N; Kg m/s ²)	p = pressure (N/m ²)
N = total molecules	N_A = Avogadro's number	\mathcal{N} = number density (N/V)
R = Gas constant = $N_A k$ (8.31446 J/Kmol)	k = Boltzmann's constant (1.38065x10 ⁻²³ J/K)	
Z = compression factor (= $\frac{V_m}{V_m^0}$)		

Gas Critical Constants

Constants	p_c	V_c	T_c
van der Waals law	$\frac{a}{27b^2}$	3b	$\frac{8a}{27bR}$
Berthelot	$\frac{1}{12} \sqrt{\frac{2aR}{3b^3}}$	3b	$\frac{2}{3} \sqrt{\frac{2a}{3bR}}$
Dieterici	$\frac{a}{4e^2 b^2}$	2b	$\frac{a}{4bR}$

Notes:

- 1 Perfect gas isotherms are obtained at high temps & high V_M
- 2 Liquids and gases coexist when attractive and repulsive forces balance
- 3 Critical constants (and Boyle temp) are related to the van der Waals constants
- 4 For $T < T_c$ isotherms oscillate

$$Z_c = \frac{3}{8} = \frac{p_c V_c}{RT_c}$$

$$\frac{dp}{dV_M} = - \frac{RT}{(V_M - b)^2} + \frac{2a}{V_M^3}$$

$$\frac{d^2p}{dV_M^2} = \frac{2RT}{(V_M - b)^3} - \frac{6a}{V_M^4}$$

$$p_r p_c = \frac{RT_r T_c}{V_r V_c - b} - \frac{a}{V_r^2 V_c^2}$$